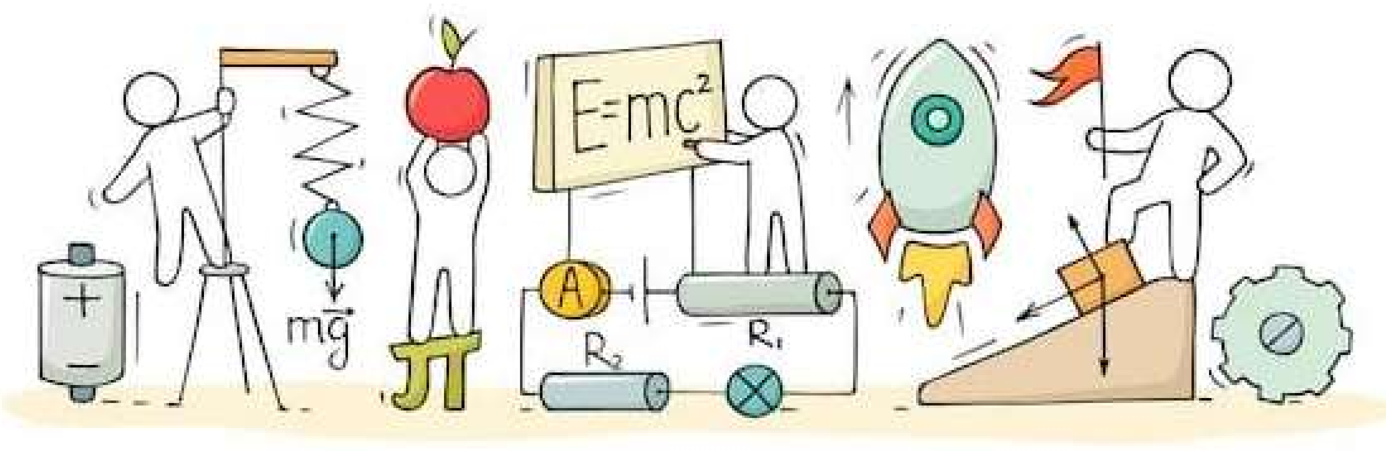
PHYSICS

Chapter 6: Work, Energy and Power



Work, Energy and Power

Top Formulae

The Scalar Product	**A.B** = AB cos θ
The scalar product follows the commutative law	**A.B = B.A**
The scalar product follows the distributive law	**A. (B + C) = A.B + A.C** λ **A. (**λ **B) =** λ **(A.B)** Where λ is a real number
Work	W = (F cos θ) d = **f·d**
Kinetic energy	$KE = \frac{1}{2}mv^2 = \frac{p^2}{2m}$
The equivalence of mass and energy	$E = m c^2$
Average power	$P_{av} = \frac{W}{t}$
Instantaneous power	$P = \frac{dW}{dt}$ P = **F·v**
Unit of power	horsepower (hp) 1 hp = 746 W 1 kWhr = 1000 (watt) × 1 (hour) = 1000 watt hour = 1 kilowatt hour (kWh) = 10^3 (W) × 3600 (s) = 3.6×10^6 J
Collisions in one dimension	$v_{1f} = \frac{(m_1 - m_2)}{m_1 + m_2} V_{1i}$ $v_{2f} = \frac{2m_1 v_{1i}}{m_1 +}$ If the two masses are equal, $v_{1f} = 0$ $v_{2f} = v_{1f}$

	If one mass dominates, e.g. $m_2 >> m_1$ $v_{1f} : -v_{1f}$ $v_{2f} : 0$
Work done by a variable force	W = $\int_{r_1}^{r_2} \vec{F}.d\vec{r}$ = Area under the F – r curve and position axis $W = \int_{x_1}^{x_2} F_x\, dx + \int_{y_1}^{y_2} F_y\, dy + \int_{z_1}^{z_2} F_z\, dz$
Work done by a spring force	$W = \frac{1}{2}kx_2^2 = \frac{1}{2}kx_1^2$ if $x_1 = 0$ and $x_2 = x$. Then $W = \frac{1}{2}kx^2$
For conservative force	$W_{AB.\,1} = W_{AB.\,2}$
For non-conservative force	$W_{AB.\,1} \neq W_{AB.\,2}$
Potential energy	(i) Of a system is always defined corresponding to a conservative internal force. (ii) Change in PE = Work done by the internal conservative force on the system $\Delta U = U_f - U_i = -W_c = -\int \vec{F_c}.d\vec{r}$
Gravitational potential energy	PE near the Earth's surface with respect to the ground = mgh
Spring potential energy	$= \frac{1}{2}kx^2$

Top Concepts

- Work done by a constant force is

  $W = Fs\cos\phi = \vec{F}.\vec{s}$
- Work done can be positive, negative or zero.
- Work done by a variable force

  $$W = \int_{s_1}^{s_2} F(s)ds$$
- Work–energy theorem: The work W done by the net force on a particle equals the change in the particle's kinetic energy.

  $$FS = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$
  $$W = K_f - K_i$$

- Gravitational potential energy does not depend on the choice of the reference surface for measuring height.

- **Gravitational potential energy:**
  (a) Energy possessed by a body changes with height with respect to the surface of the Earth.
  (b) $GPE = -W_{\text{Gravitational Force}}$

- Law of conservation of mechanical energy:
  Total mechanical energy of the system always remains constant in the absence of dissipative forces.

- Total mechanical energy of the system equals the sum of potential energy and kinetic energy.

- Work done on a system by conservative forces implies that the mechanical energy of the system remains constant.

- Work done by the conservative force is the same along any path.

- Total work done by the gravitational force on the body moving along a closed loop is always zero.

  W_{gravity} in closed loop = Zero

- Work done on a system by non-conservative forces implies that the mechanical energy of the system is not conserved.

- A conservative force is the negative gradient of potential energy function.
  $F(x) = -\Delta U/\Delta x$

- Power is the rate at which work is done or energy is transformed.

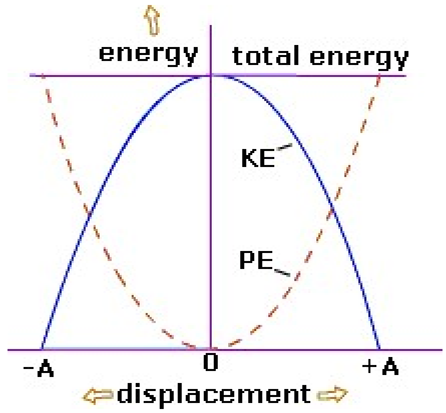
- The unit of power is watt.

  1 watt = 1 joule/second
- Linear momentum of an isolated system is always conserved in a collision.

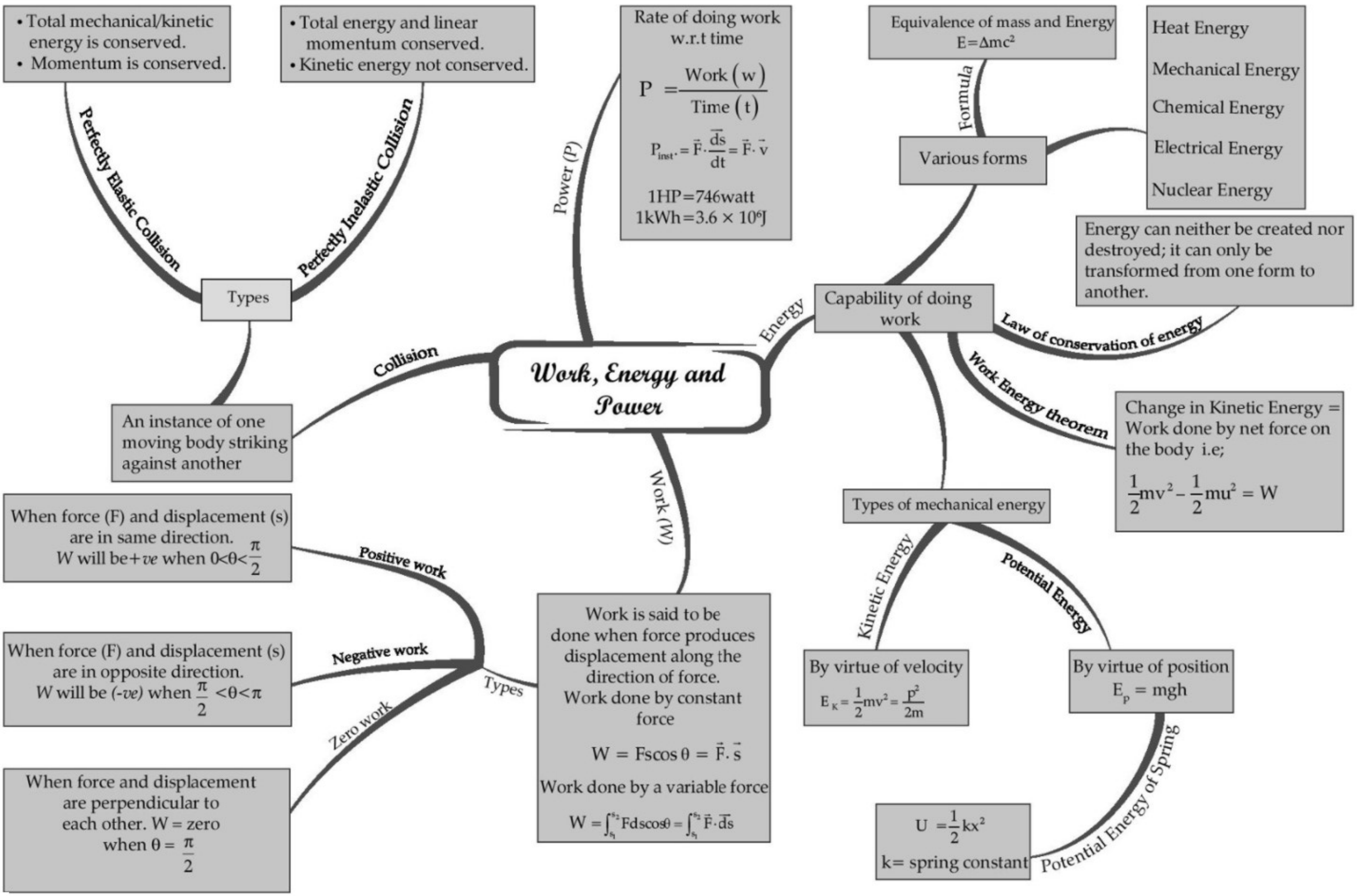
- A collision in which the total kinetic energy of the system is conserved is called elastic.
  A collision in which the total kinetic energy of the system is not conserved is called inelastic.
- When two bodies collide, stick together and have a common final velocity, the collision is completely inelastic.

Diagrams

Energy curve


energy
total energy
KE
PE
-A
0
+A
displacement

CHAPTER - 6 : WORK, ENERGY AND POWER



Important Questions

Multiple Choice questions-

1. When a force of 50 N acts on a body, the body is displaced through a distance of 3 m in a direction normal to the direction of the force. The work done by the force

(a) 150 J

(b) 1470 J

(c) Zero

(d) -150 J

2. A body of mass 20 kg is initially at a height of 3 m above the ground. It is lifted to a height of 2 m from that position. Its increase in potential energy is

(a) 100 J

(b) 392 J

(c) 60 J

(d) -100 J

3. A wooden cube having mass 10 kg is dropped from the top of a building. After 1 s, a bullet of mass 20 g fired at it from the ground hits the block with a velocity of 1000 m/s at an angle of 30° to the horizontal moving upwards and gets imbedded in the block. The velocity of the block/bullet system immediately after the collision is

(a) 17 m/s

(b) 27 m/s

(c) 52 m/s

(d) 10 m/s

4. A body of mass 10 kg is moved parallel to the ground, through a distance of 2 m. The work done against gravitational force is

(a) 196 J

(b) -196 J

(c) 20 J

(d) zero

5. A quantity of work of 1000 J is done in 2 seconds. The power utilized is

(a) 998 W

(b) 1002 W

(c) 2000 W

(d) 500 W

6. A body of mass 1 kg travels with a velocity of 10 m/s, this a wall and rebounds. If 50% of its initial energy is wasted as heat, its kinetic energy at the instant of rebounding is

(a) 20 J

(b) 60 J

(c) 50 J

(d) 25 J

7. A marble moving with some velocity collides perfectly elastically head-on with another marble at rest having mass 1.5 times the mass of the colliding marble. The percentage of kinetic energy by the colliding marble after the collision is

(a) 4

(b) 25

(c) 44

(d) 67

8. A particle of mass m is moving in a horizontal circle of radius r under a centripetal force given by $(-kr^2)$ where k is a constant , then

(a) the total energy of the particle is (- k/2r)

(b) the kinetic energy of the particle is (k/r)

(c) the potential energy of the particle is (k/2r)

(d) the kinetic energy of the particle is (- k/r)

9. A sphere of mass m moving with a constant velocity u hits another stationary sphere of the same mass. If e is the coefficient of restitution, then the ratio of velocity of the two spheres after the collision will be

(a) 1– e / 1 + e

(b) 1 + e / 1 – e

(c) e + 1 / e – 1

(d) e – 1 / e + 1

10. Two masses 1 g and 4 g are moving with equal kinetic energies.s The ratio of the magnitudes of their linear momenta is

(a) 4 : 1

(b) 0 : 1

(c) 1 : 2

(d) 1 : 6

Very Short:

1. What is the source of the kinetic energy of the falling raindrops?
2. A spring is stretched. Is the work done by the stretching force positive or negative?
3. What is the type of collision when?

   (a) Does a negatively charged body collide with a positively charged body?

   (b) Do macroscopic bodies collide?

   (c) Do two quartz balls collide?
4. (a) Give two examples of potential energy other than gravitational potential energy.

   (b) Give an example of a device that converts chemical energy into electrical energy.

   (c) Heat energy is converted into which type of energy in a steam engine?

   (d) Where is the speed of the swinging pendulum maximum?

   (e) A heavy stone is lowered to the ground. Is the work done by the applied force positive or negative?
5. What is the work done by the centripetal force? Why?
6. (a) What is the work done by the tension in the string of simple pendulum?

   (b) What is the work done by a porter against the force of gravity when he is carrying a load on his hand and walking on a horizontal platform?

   (c) Name the force against which the porter in part (A) is doing some work.
7. When an arrow is shot, wherefrom the arrow will acquire its K.E.?
8. When is the exchange of energy maximum during an elastic collision?
9. Does the work done in raising a load onto a platform depend upon how fast it is raised?
10. Name the parameter which is a measure of the degree of elasticity of a body.

Short Questions:

1. An airplane's velocity is doubled,

   (a) What happens to its momentum? Is the law of conservation of momentum obeyed?

   (b) What happens to its kinetic energy? Is the law of conservation of energy obeyed?
2. In a thermal station, coal is used for the generation of electricity. Mention how energy changes from one form to the other. before it is transformed into electrical energy?

3. Chemical, gravitational and nuclear energies are nothing but potential energies for different types of forces in nature. Explain this statement clearly with examples.
4. What went wrong at the Soviet atomic power station at Chernobyl?
5. A man can jump higher on the moon than on Earth. With the same effort can a runner improve his timing for a 100 m race on the moon as compared to that on Earth?
6. How many MeV are there in a 1-watt hour?
7. What is Newton's experimental law of impact?
8. Two masses one n times as heavy as the other have the same K.E. What is the ratio of their momenta?

Long Questions:

1. (a) State work-energy theorem or principle.

   (b) State and prove the law of conservation of energy.

Assertion Reason Questions:

1. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
   (c) Assertion is correct, reason is incorrect
   (d) Assertion is incorrect, reason is correct.

Assertion: A work done by friction is always negative.
Reason: If frictional force acts on a body its K.E. may decrease.

2. **Directions:** Each of these questions contain two statements, Assertion and Reason. Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

   (a) Assertion is correct, reason is correct; reason is a correct explanation for assertion.
   (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion
   (c) Assertion is correct, reason is incorrect
   (d) Assertion is incorrect, reason is correct.

Assertion: The work done in moving a body over a closed loop is zero for every force in nature.
Reason: Work done depends on nature of force.

Case Study Questions:

1. The scalar product or dot product of any two vectors A and B, denoted as A.B (read A dot B) is defined as

   A.B = A B cos θ

   Where q is the angle between the two vectors. Since A, B and cos θ are scalars, the dot product of A and B is a scalar quantity. Each vector, A and B, has a direction but their scalar product does not have a direction. Following are properties of dot product

   - the scalar product follows the commutative law: A.B = B.A
   - Scalar product obeys the distributive law: (B + C) = A.B + A.C Further, A. (λ B) = λ (A.B) where λ is a real number.
   - For unit vectors i, j, k we have

   i × i = j × j = k × k = 1 and i × j = j × k = k × i = 0

   - A x A = |A | |A| cos 0 = A^2.
   - B = 0, if A and B are perpendicular.

The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement. Thus

W = (F cos θ) d = F.d (We see that if there is no displacement, there is no work done even if the force is large. Work has only magnitude and no direction. Its SI unit is (N m) or joule (J). Thus, when you push hard against a rigid brick wall, the force you exert on the wall does not work.

No work is done if:

- The displacement is zero.
- The force is zero. A block moving on a smooth horizontal table is not acted upon by Horizontal force (since there is no friction) but may undergo a large displacement.
- The force and displacement are mutually perpendicular. This is so since, for θ = π/2 rad
- Cos (π/2) = 0. For the block moving on a smooth horizontal table, the gravitational force mg does no work since it acts at right angles to the displacement. If we assume that the moon's orbits around the earth are perfectly circular, then the earth's gravitational force does not work. The moon's instantaneous displacement is tangential while the earth's force is radially inwards and θ = π/2.

   i. Scalar product A.B = B.A is

      a. Commutative law

b. Distributive law
c. Both a and b
d. None of these

ii. When force acts in the direction of displacement then work done will be
a. Positive
b. Negative
c. Both a and b can possible
d. None of these

iii. Define scalar product. give its properties

iv. Define work done. Give its SI unit

v. Write down the conditions for which work done is zero

2. The kinetic energy possessed by an object of mass, m and moving with a uniform velocity, v is

$$K = \frac{1}{2} * mv^2 = \frac{1}{2} \, v.v$$

Kinetic energy is a scalar quantity. The kinetic energy of an object is a measure of the work and the energy possessed by an object is thus measured in terms of its capacity of doing work. The unit of energy is, therefore, the same as that of work, that is, joule (J).

Work energy theorem: The change in kinetic energy of a particle is equal to the work done on it by the net force. Mathematically

$$K_f - K_i = W$$

Where K_i and K_f are respectively the initial and final kinetic energies of the object. Work refers to the force and the displacement over which it acts. Work is done by a force on the body over a certain displacement.

i. Kinetic energy is
a. Scalar quantity
b. Vector quantity
c. None of these

ii. Which of the following has same unit?
a. Potential energy and work
b. Kinetic energy and work
c. Force and weight

d. All of the above

iii. What is work energy theorem?

iv. Kinetic energy is scalar quantity. Justify the statement.

v. Give formula for kinetic energy of body.

✓ Answer Key:

Multiple Choice Answers-

1. Answer: (c) Zero
2. Answer: (b) 392 J
3. Answer: (a) 17 m/s
4. Answer: (d) zero
5. Answer: (d) 500 W
6. Answer: (d) 25 J
7. Answer: (a) 4
8. Answer: (a) the total energy of the particle is (- k/2r)
9. Answer: (a) 1 – e / 1 + e
10. Answer: (c) 1 : 2

Very Short Answers:

1. Answer: It is the gravitational potential energy that is converted into kinetic energy.
2. Answer: Positive because the force and the displacement are in the same direction.
3. Answer:

   (a) Perfectlyjnelastic collision.

   (b) Inelastic collision.

   (c) Perfectly elastic collision.
4. Answer:

   (a) Electrostatic P.E. and elastic P.E.

   (b) Daniell cell.

   (c) Mechanical energy.

   (d) At the bottom of the swing.

   (e) Negative work.
5. Answer: Zero. This is because the centripetal is always perpendicular to the

displacement.

6. Answer:

   (a) zero

   (b) zero

   (c) Frictional force.

7. Answer: It is the potential energy of the bent bow which is converted into K.E.
8. Answer: When two colliding bodies are of the same mass, there will be a maximum exchange of energy.
9. Answer: The work done is independent of time.
10. Answer: Coefficient of restitution.

Short Questions Answers:

1. Answer:

   (a) The momentum of the airplane will be doubled. Yes, the law of conservation of momentum will also be obeyed

   because the increase in momentum of the airplane is simultaneously accompanied by an increase in momentum of exhaust gases.

   (b) K.E. becomes four times. Yes, the law of conservation of energy is obeyed with the increase in K.E. coming from the chemical energy of fuel i. e. from the burning of its fuel.

2. Answer: When coal is burnt, heat energy is produced which converts water into steam. This steam rotates the turbine and thus heat energy is converted into mechanical energy of rotation. The generator converts this mechanical energy into electrical energy.
3. Answer: A system of particles has potential energy when these particles are held a certain distance apart against some force. For example, chemical energy is due to the chemical bonding between the atoms. Gravitational energy arises when the objects are held at some distance against the gravitational attraction.

   Nuclear energy arises due to the nuclear force acting between the nuclear particles.

4. Answer: In this reactor, graphite was used as a moderator. The fuel elements were cooled by water and steam was produced from within the reactor. Both water and the steam came in contact with hot graphite. Due to this hydrogen and carbon-monoxide (CO) were released. When they came in contact with air, there was a big explosion.
5. Answer: Man can jump higher on the moon because the acceleration due to gravity on the moon is less than that on the Earth. But acceleration due to gravity does not

affect the horizontal motion. Hence the runner can't improve his timing on the moon for the 100 m race.

6. Answer:

We know that 1 watt hour = 1 JS^{-1} × 3600 s = 3600 J

Also we know that $1\ eV = 1.6 \times 10^{-19}\ J$

or $1\ J = \frac{1}{1.6\times10^{-19}}\ eV$

$\therefore$ $1\text{ watt hour} = 3600 \times \frac{1}{1.6\times10^{-19}}\ eV$

$= 2.25 \times 10^{22}\ eV$

Now $1\ MeV = 10^{6}\ eV$

or $1\ eV = 10^{-6}\ MeV$

$1\text{ watt hour} = 2.25 \times 10^{22} \times 10^{-6}\ MeV$

$= 2.25 \times 10^{16}\ MeV.$

7. Answer: The ratio of the relative speed of separation after a collision to the relative speed of approach before the collision is always constant. This constant is known as the coefficient of restitution. It is denoted by e.

$\therefore e = \frac{V_{2f} - v_{1f}}{u_{1i} - u_{2i}}$

where u_{1i} and u_{2i}, are the velocities of the bodies before collision and v_{2f}, v_{1f} are the velocities of the bodies after the collision.

8. Answer:

We know that $p = \sqrt{2mE_k}$ or $E_k = \frac{p^2}{2m}$

Since E_k is constant

$\therefore$ $p \ \alpha \ \sqrt{m}$

$\therefore$ $p_1 \ \alpha \ \sqrt{nm}$ and $p_2 \ \alpha \ \sqrt{m}$

$\therefore$ $\frac{p_1}{p_2} = \frac{\sqrt{nm}}{\sqrt{m}} = \frac{\sqrt{n}}{1}.$

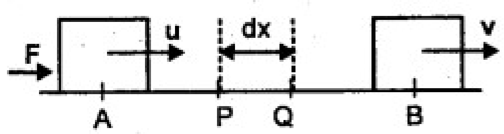
Long Questions Answers:

1. Answer:

(a) It states that the work done on a body is equal to the change in its kinetic energy.

i.e., W = change in kinetic energy

Proof: Let m = mass of a body moving in a straight line with a constant initial velocity u.



Let F = force applied on it at point A to B so that its velocity is V at B.

If dx = small displacement from P to Q

and a = acceleration produced in the body, then

F = ma

If dw be the work done from P to Q, then

$$dw = \mathbf{F}.\mathbf{dx} = F\,dx\cos 0$$
$$= ma\,dx = m\frac{dv}{dt}dx$$
$$= mdv.\left(\frac{dx}{dt}\right) = mv.dv$$

If W = total work done from A to B, then

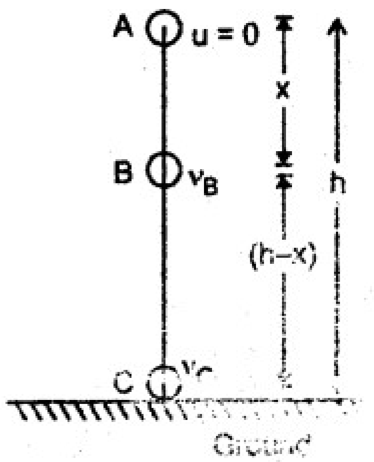
$$W = \int_u^v dW = m\int_u^v u\,dv$$

$$m\left[\frac{v^2}{2}\right]_u^v = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$
$$= \text{change in K.E. of body.}$$

(b) It states that energy can neither be created nor can be destroyed but it can be changed from one form of energy into another i.e. total energy = constant.

Proof: Let a body of mass m be lying at rest at point A at a height h above the ground. Let it be allowed to fall freely and reaches a point B after falling through a distance x and it finally hits the ground at point C. Let v and V be its velocities at points B and C respectively.

∴ AB = x and BC = h – x



At point A: u = 0

∴ K.E. = 0

P.E. = mgh

If E be the total energy of the body, then

E = K.E. + P.E. = 0 + mgh

or

E = mgh …. (i)

At point B: using the relation,

$v^2 - u^2 = 2as$, we get

$v_B^2 - 0 = 2gx$

($\because$ here $v = v_B$, $u = 0$, $a = g$, $s = x$)

or $v_B^2 = 2gx$

∴ $K.E. = \frac{1}{2}mv_B^2 = \frac{1}{2}m \times (2gx) = mgx$

and $P.E. = mg(h - x)$

∴ $E = K.E. + P.E. = mgx + mgh - mgx$

or $E = mgh$ .... (*ii*)

At point C: Here, $v = v_C$, $a = g$, $s = h$

$$\therefore \quad v_C^2 - 0^2 = 2gh$$

$$v_C^2 = 2gh$$

Thus $$K.E. = \frac{1}{2}mv_C^2 = \frac{1}{2} m \times 2gh = mgh$$

and $$P.E. = mg(0)$$

$(\because$ height from ground at point $C = 0)$

$$= 0$$

$$\therefore \quad E = K.E. + P.E. = mgh + 0 = mgh$$

or $$E = mgh \quad \text{.... } (iii)$$

Thus, from (i), (ii), and (iii), it is clear that total energy at points A, B, and C is the same. It is purely P.E. at A and purely K.E. at point C.

Assertion Reason Answer:

1. (d) Assertion is incorrect, reason is correct.

Explanation

When frictional force is opposite to velocity, kinetic energy will decrease.

2. (b) Assertion is correct, reason is correct; reason is not a correct explanation for assertion

Explanation:

In close loop, s = 0, and so W = Fs = 0.

Case Study Answer:

1. **i.** (a) commutative law

ii. (a) positive

iii. the scalar product or dot product of any two vectors A and B, denoted as A.B (read A dot B) is defined as

A.B = A B cos q. where q is the angle between the two vectors. Since A, B and cos θ are scalars, the dot product of A and B is a scalar quantity. Each vector, A and B, has a direction but their scalar product does not have a direction. Following are properties of dot product

- the scalar product follows the commutative law:

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- Scalar product obeys the distributive law:

  (B + C) = A.B + A.C

Further, A. (λ B) = λ (A.B) where λ is a real number.

- For unit vectors i, j, k we have

  i × i = j × j = k × k = 1 and i × j = j × k = k × i = 0
- A×A = |A ||A| cos 0 = A^2.
- B = 0, if A and B are perpendicular.

Iv. The work done by the force is defined to be the product of component of the force in the direction of the displacement and the magnitude of this displacement. Thus

W = (F cos θ) d = F.d

Work has only magnitude and no direction. Its SI unit is (N m) or joule (J).

v. No work is done if:

- The displacement is zero.
- The force is zero. A block moving on a smooth horizontal table is not acted upon by a Horizontal force (since there is no friction) but may undergo a large displacement.
- The force and displacement are mutually perpendicular. This is so since, for θ = π/2 rad Cos (π/2) = 0

2. **i.** (a) Scalar quantity

ii. (c)All of the above

iii. **Work energy theorem:** The change in kinetic energy of a particle is equal to the work done on it by the net force. Mathematically

$K_f – K_i$ = W

Where K_i and K_f are respectively the initial and final kinetic energies of the object. Work refers to the force and the displacement over which it acts. Work is done by a force on the body over a certain displacement. Energy possessed by object due to its motion is called as kinetic energy. Its SI unit is N-m or Joule (J).

iv. Kinetic energy is scalar quantity as it is a work done and work done is scalar quantity hence kinetic energy is also scalar quantity and doesn't have any direction.

v. the kinetic energy possessed by an object of mass, m and moving with a uniform velocity, v is

$$K = \frac{1}{2}*mv^2 = \frac{1}{2}\ v.v$$

Kinetic energy is a scalar quantity. Having unit, the same as that of work, that is, joule (J).